# A Gm-C Filter Bank for Supraharmonic Analysis Application

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Abstract—The components superimposed to the electrical power grid signal in the range of 2 to 150 kHz are called supraharmonics. A technique for analyzing these signals based on subsampling was proposed in the literature, which uses a bank of passband analog filters in order to decompose the supraharmonic spectrum into ranges of 15 kHz. This paper proposes a Gm-C filter bank devised to perform this task, suitable for integrated circuit implementation. The design of one of the filters is briefly described, as well as simulation results.

*Keywords—power quality, supraharmonic, analog filter bank, Gm-C filters.* 

## I. INTRODUCTION

The electrical power grid is designed to transfer energy at 50 or 60 Hz. However, components of higher frequencies may appear in the power system [1]. Recently, there is a growing concern about the distortion in the range of 2 kHz to 150 kHz. The components in this frequency range are called supraharmonics, and are generated mainly by the introduction of distributed generation equipment, based on inverters that employ Pulse Width Modulation (PWM) techniques for power conversion. The supraharmonics are a possible cause of problems due to their interaction with other kind of devices, such as power line communications (PLC) modems connected in the network [1]-[2].

Two of the international standards that describe the measurement methodology for supraharmonics components are the IEC 61000-4-30 [3], which describes a measurement method for supraharmonics in the range of 9 kHz to 150 kHz, and the IEC 61000-4-7 [4], focused in the range of 2 kHz to 9 kHz. These standards suggest the use of a sampling rate at 1024 kHz, which is a high sampling rate in the context of power quality analyzers. The technique suggested to calculate the supraharmonics components is the Fast Fourier Transform (FFT). This value of sampling rate will result in large windows of digital signals and thus a high computational cost to process all this data.

In order to overcome these limitations, the authors proposed in [5] the usage of an analog filter bank associated with subsampling and other digital signal processing techniques in order to reduce the amount of data and the computational burden of the calculation of the supraharmonics components by FFT.

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Although the original project proposed a passive bank of high-order filters with discrete components, some issues motivated the research on an integrated filter bank for this application. Among several approaches for continuous-time integrated filters [6], the Gm-C filters, based on the operational transconductance amplifier (OTA) were chosen, especially due to their tunability.

This paper proposes a Gm-C filter bank devised to perform the supraharmonic analysis technique, which is suitable for integrated implementation. The design of the filter is briefly described, with a design example for one of the frequency ranges. Simulations results are also shown.

The work is divided as follows: in the Section II, the supraharmonic measurement is briefly described. The filter topology is presented in Section III, as well as a design example. Section IV shows simulation results. Finally, a conclusion is presented in Section V.

### **II. OVERALL SYSTEM DESCRIPTION**

The supraharmonics measuring system is illustrated in Fig. 1. The grid voltage or current (in this case, converted into a voltage signal), after being conditioned, is applied to the system (the *Input* signal in figure). The lower frequency components (below 2 kHz) are filtered by a high-pass (HP) filter, and frequencies above the maximum supraharmonic frequency (150 kHz) are filtered by a low-pass (LP) filter.

The filter bank is composed by ten passband filters, each with a bandwidth of 15 kHz, covering the entire range of the supraharmonic spectrum (2 kHz to 150 kHz). The exception is the first filter, which has a bandwidth of 13 kHz (from 2 kHz to 15 kHz). The required filters have elliptic characteristics and present 45 dB of attenuation in the stopband. Two adjacent filter bands have 500 Hz of overlap.

The outputs of the filter bank are connected to an analog multiplexer (MUX) that is used to select which filter output to analyze and an Analog to Digital Converter (ADC) is used to digitize the signal.

Without the use of the filter bank, the sampling rate,  $F_s$ , needed to properly analyze the signal in digital domain, according with the Shannon–Nyquist theorem [7], would be very high (1024 kHz). However, since the filter bank limits the bandwidth of the signal at the input of the ADC, the extended Nyquist criterion [7] can be applied:



Fig. 1. The filter bank inserted in the measuring system.

$$F_S \ge 2\Delta f \tag{1}$$

where  $F_S$  is the sampling frequency and  $\Delta f$  is the bandwidth of the filter. Thus, the sampling frequency can be reduced to 32 kHz.

The original filter bank was implemented by a passive LC ladder network, terminated with a resistor, as represented in Fig 2. The circuits are buffered at the input and the output by operational amplifiers. The complete circuit can be found in [8]. Taking into account that the bank is composed by ten 16<sup>th</sup>-order passband filters, a considerable amount of space on the printed circuit board is needed for the filters using discrete parts, even by employing surface mounted device (SMD) components.

Moreover, the exact values calculated for the inductors and capacitors could not be employed, and approximated commercial values were applied. This leads to a considerable difference in the filter's frequency response from the desired response, which could impair the system performance.

These factors motivated the design of the integrated filter bank, which is described in the following section. The HP and LP filters that condition the signal before being applied to the bank, as well as the analog MUX and the ADC, are out of the scope of this work.

### **III. THE PROPOSED FILTER BANK**

To design the filters and obtain their respective transfer functions, the MATLAB function 'ellipord' has been used. This function calculates the lowest order elliptic filter that follows the specifications defined by the user. For the passband filter, these specifications are the passband and stopband edge frequencies (radians/second), the maximum passband ripple (in dB) and the minimum stopband attenuation (in dB). For the desired parameters shown in section II, 16<sup>th</sup>-order transfer functions were obtained.



Fig. 2. Representation of the original filter based on a LC ladder, terminated with a a resistor.

# A. Filter Topology

The Gm-C filters are based on the operational transconductance amplifier (OTA), so they are also called OTA-C filters. Their basic block is the Gm-C integrator, composed by an OTA (with transconductance  $g_m$ ) and an output capacitor, C, as depicted in Fig. 3. The integrator time constant,  $\tau$ , is given by:

$$\tau = \frac{c}{g_m} \tag{2}$$

One of the features of the Gm-C filters is their tunability, since the OTA transconductance can be adjusted, what can be done, in general, by adjusting the OTA biasing current.

The implementation of high-order Gm-C filters is not usual, but in the literature, it is possible to find several second-order structures called biquad filters. This greater ease and variety of second-order filters suggested the decomposing of the  $16^{\text{th}}$ -order transfer functions into eight cascaded second-order sections, as shown in Fig. 4. This can be done in MATLAB using the function 'tf2sos', which finds all the second-order sections that, when multiplied by each other, result in the original high-order transfer function.

Different topologies presented in the literature have been evaluated and the topology that had better results was the one introduced in [9]. This structure is called follow-the-leaderfeedback (FLF) Gm-C filter. A second-order FLF filter structure is shown in Fig. 5. It is composed by two Gm-C integrators, a feedback network (upper part) and a feedforward output summation network (lower part).

Although it is a non-canonical structure, with a higher number of OTAs than it is present in other topologies [10], this circuit has some advantages, as more degrees of freedom for design. This is necessary to implement the transfer functions of the elliptical filters. Moreover, the FLF filter has only grounded capacitors and has the possibility of achieving a zero capacitor spread, which is interesting for an integrated implementation of a high-order filter.



Fig. 3. The Gm-C integrator.



Fig. 4. Cascade of eight second-order sections.



Fig. 5. A second-order FLF Gm-C filter.

It is worth mentioning that the chosen topology also allows the creation of higher order filters. However, after evaluating three different strategies (the complete 16<sup>th</sup>-order filter, the cascade of four 4<sup>th</sup>-order stages and the cascade of eight second-order stages), it was observed that the secondorder approach has lower sensitivity and better quality in the frequency response. The sensitivity tests performed were the Monte Carlo method and the Gaussian variation of the gm values of the filters. With these tests the use of second order blocks still proved to be advantageous, and it was the strategy used in the work.

## B. Filter Design

Given a generic second order transfer function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{A_2 * s^2 + A_1 * s + A_0}{B_2 * s^2 + B_1 * s + B_0}$$
(3)

According to [9], the equations for the second-order FLF filter in Fig. 5 are:

$$\tau_j = \frac{c_j}{g_j} \; ; \; f_j = \frac{g_{f_j}}{g_{f_r}} \; \; (j = 1, 2) \tag{4, 5}$$

$$a_j = \frac{g_{a_j}}{g_{a_r}}$$
 (j = 0, 1, 2) (6)

$$B_o = f_2 \tag{7}$$

$$B_1 = f_1 * \tau_2 \tag{8}$$

$$B_2 = \tau_1 * \tau_2 \tag{9}$$

$$A_o = a_0 * f_2 + a_2 \tag{10}$$

$$A_1 = (a_0 * f_1 + a_1) * \tau_2 \tag{11}$$

$$A_2 = a_0 * \tau_1 * \tau_2 \tag{12}$$

Using these equations and comparing them with the desired transfer functions, it is possible to find all the transconductance values of the filter. The values of  $g_{ar}$ ,  $g_{fr}$ ,  $g_1$ ,  $g_2$ ,  $C_1$  and  $C_2$  are arbitrary.

# C. Design example

The design of one passband filter for a specific frequency range of the bank is briefly illustrated here. The  $16^{\text{th}}$ -order transfer function for the 45-60 kHz passband filter, H(s), can be decomposed in eight second-order transfer functions, so that:

$$H(s) = H_1(s) * H_2(s) * \dots * H_8(s)$$
(13)

Where:

$$H_1(s) = \frac{0.5233s^2 - 7.615*10^{-12}s + 2.342*10^{10}}{s^2 + 789.4s + 1.421*10^{11}}$$
(14)

$$H_2(s) = \frac{0.5233s^2 - 2.38*10^{-13}s + 1.328*10^{11}}{s^2 + 3909s + 1.406*10^{11}} \quad (15)$$

$$H_3(s) = \frac{0.5233s^2 + 8.148*10^{10}}{s^2 + 1.31*10^4 s + 1.348*10^{11}}$$
(16)

$$H_4(s) = \frac{0.5233s^2 + 6.092*10^{-11}s + 7.627*10^{10}}{s^2 + 2.85*10^4 s + 1.184*10^{11}}$$
(17)

$$H_5(s) = \frac{0.5233s^2 - 1.218*10^{-10}s + 7.521*10^{10}}{s^2 + 2.566*10^4 s + 9.596*10^{10}}$$
(18)

$$H_6(s) = \frac{0.5233s^2 - 2.285*10^{-11}s + 3.819*10^{10}}{s^2 + 1.036*10^4 s + 8.43*10^{10}}$$
(19)

$$H_7(s) = \frac{0.5233s^2 + 5.711*10^{-12}s + 4.079*10^{10}}{s^2 + 2963s + 8.081*10^{10}}$$
(20)

$$H_8(s) = \frac{0.5233s^2 + 3.808 * 10^{-12} s + 4.137 * 10^{10}}{s^2 + 592.1s + 7.995 * 10^{10}}$$
(21)

The transconductance values obtained for each secondorder section (referring to Fig. 4) are shown in Table I. The parameters for all the filters of the bank can be found in [11].

TABLE I.PARAMETERS FOR THE 45–60 KHz BAND PASS FILTER

Section	Transconductances (in µA/V)					
	$g_1$	$g_2$	$g_{fl}$	$g_{f^2}$	<b>g</b> a1	$g_{a2}$
S1	0.1	1	0.0789	14.2	-0.0413	-5.1
S2	0.1	1	0.39	14.1	-0.204	5.92
S3	0.1	1	1.31	13.5	-0.685	1.1
S4	0.1	1	2.85	11.8	-1.49	1.43
S5	0.1	1	2.56	9.59	-1.34	2.5
S6	0.1	1	1.04	8.43	-0.542	-0.592
S7	0.1	1	0.296	8.08	-0.155	-0.149
S8	0.1	1	0.0592	8	-0.031	-0.047
For all sections			$g_{a0} = 5.23 \ \mu A/V$			
			$g_{ar} = g_{fr} = 10 \ \mu A/V$			
			$C_1 = C_2 = 1 \text{ pF}$			

From a practical point of view, the values chosen for the capacitors (1 pF) are adequate for on-chip implementation. For the transconductances, it can be noted that some values are of 100 nA/V or even lower. The implementation of OTAs with transconductances of such orders of magnitude would

require some special design technique, as the ones discussed in [12].

#### IV. SIMULATION RESULTS

The filter bank was simulated in LTSpice XVII using a simplified model for the OTAs, consisting of a voltage-controlled current source. For the specific range of 45-60 kHz, the results shown in Fig. 6 compare the ideal frequency response (obtained from the transfer function) with the passive filter with discrete parts (considering both the exact values and the commercial values of the components), and with the proposed Gm-C filter.

It can be observed that the use of components with commercial values degrades the response of the passive filter. By the other hand, the Gm-C active filter, if compared to the passive filter, presents less passband ripple. Moreover, it is capable of reaching the required 45 dB (or more) of attenuation over both stopbands, while the passive filter gives this attenuation only near the cutoff frequencies.

The frequency responses for the whole bank are shown in Fig. 7.



Fig. 6. Comparison of different topologies for the 45-60 kHz range.



V. CONCLUSION

Supraharmonics are power quality disturbances that have aroused growing interest in the electrical power systems context. This paper proposed a Gm-C filter bank devised to be applied on a supraharmonic analyzer. The circuit, composed by ten passband filters, is suitable for implementation on an integrated circuit.

The filters are based on the FLF topology, which has more OTAs than other Gm-C circuits. However, it has more degrees of freedom for design, thus allowing the design of the elliptical filters. Each of the ten passband filters are composed by a cascade of eight second-order FLF sections.

Besides the possibility of integration, the tunability of the Gm-C filters can lead to frequency responses closer to the ideal, if compared to the original passive filters with discrete components, as proven by simulation results.

There are several factors that must be considered when dealing with the design of an actual Gm-C filter, especially referring to the OTAs, as linearity, dynamic range, noise and output impedance, between others. Therefore, the design of the proposed filter bank on a submicrometer CMOS technology is proposed as a future work.

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Fig. 7. Frequency responses of the bank.